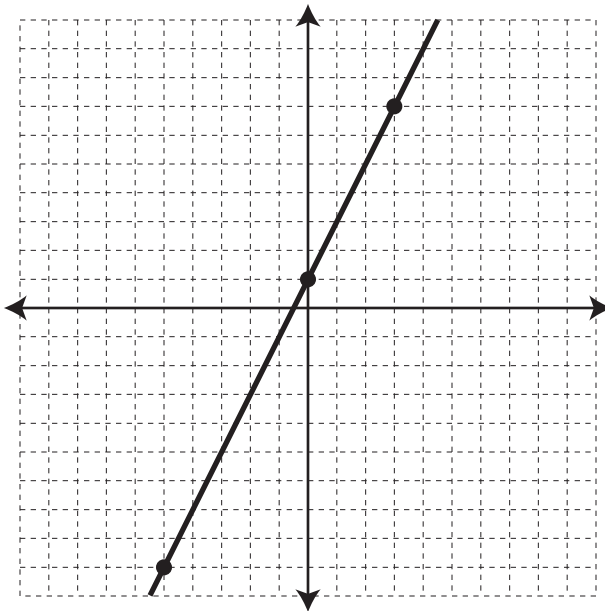


Equations of Lines

In the last class, I went over some of the properties of lines out loud, asking you not to take notes. Here is a handout that contains similar information.

How an equation turns into a line

When we graph a line, it connects all the points described by a certain relationship between “x” and “y”. For example, the equation $y = 2x + 1$ is a relationship where the y-value is one more than double the x-value. Some examples of pairs of numbers where this is true are $(3,7)$, or $(-5,-9)$ or $(0,1)$. If we plot these points and connect them with a line, we have graphed the equation $y = 2x + 1$.

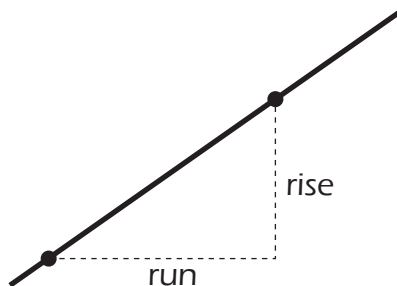


Remember, any point on this line has a y-value that is one more than double its x-value.

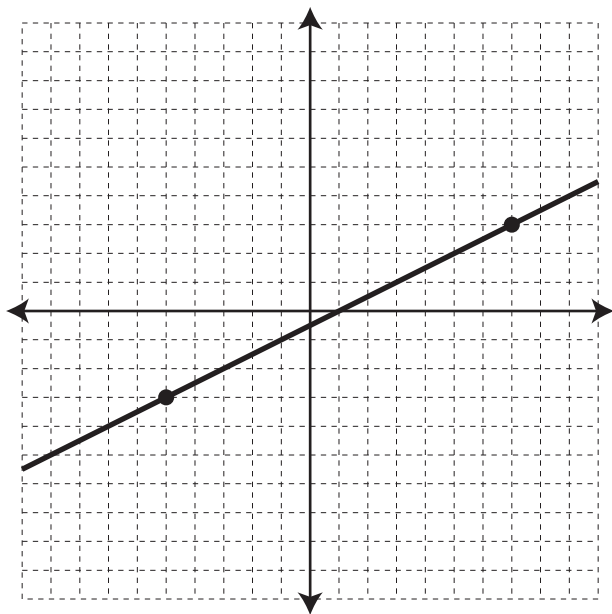
Question: On this line, if the x-value is 3.41, what is the y-value?

The Slope of a Line

The slope of a line is its steepness. It is a number that results from the ratio between the vertical distance and horizontal distance between two points on a line. These distances are commonly referred to as the “rise” and the “run”.



If you know the coordinates of two points on the line, you can figure out the slope by doing some simple calculations. For example, suppose on a new line (different from the one on page 1) there are the points $(-5, -3)$ and $(7, 3)$.



The rise between these two points is the vertical distance between them. To figure out this vertical distance, we need to look at the y-values, because the y-values tell us how far the points are up or down from the x-axis, which has a y-value of zero.

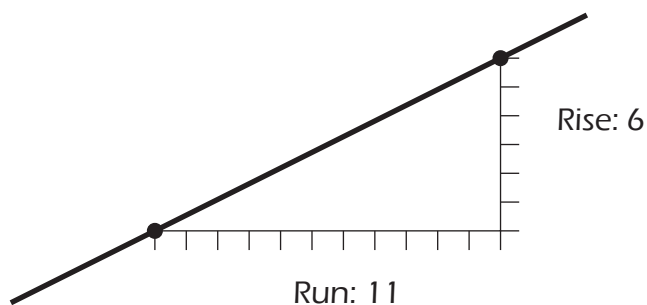
$$\text{Rise: } 3 - (-3) = 6$$

The vertical distance between the two points is seven. Where did the numbers 3 and -3 come from?

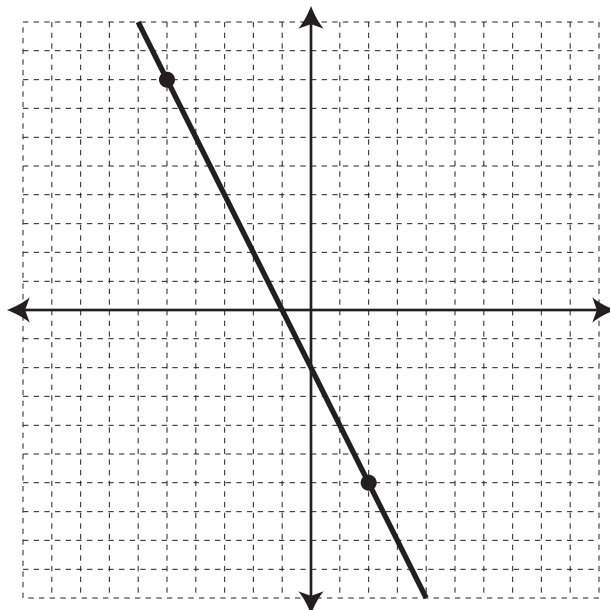
Now, in the space below, use the x-values to calculate the run between these two points.

Since the slope is a ratio of the rise to the run, we can write it in many forms. We could write in words that the slope is "six to eleven". We could also write it as $6:11$. However, the most common way to write slope is as a fraction with the rise above the run. Write the slope of this line in the space below.

If we look at this line without the background grid, you can see the rise and run:



One thing we need to be careful of when finding the slope is whether it is positive or negative. We consider a slope to be positive if the line points **up** to the **right**, and negative if the line points **down** to the **right**. If you don't have a picture of the line in front of you, there is one simple thing you can do to make sure your calculation of slope is correct: make sure that you subtract in the same order. For example:



The two points on this graph are $(-5, 8)$ and $(2, -6)$. To find the rise between them we could subtract $8 - (-6) = 14$. To find the run, we must subtract the x-values **in the same order**. That means that the run must be $(-5) - 2 = -7$, (NOT $2 - (-5)$!). In this way, we calculate the slope to be:

$$\frac{\text{rise}}{\text{run}} = \frac{14}{-7} = -2$$

If you subtract the x- and y- values in a different order, you will get the right slope, but the sign will be wrong.

Graphing from an equation without making a table of values

Two classes ago we graphed equations by finding examples of points and drawing a line through them. It turns out that there are patterns we can use to graph lines without having to use a table of values.

In the last class we discovered that in the equation of lines, the number that "x" is multiplied by is actually the slope of the line. For example, in the equation $y = 3x - 1$ the slope of the line will

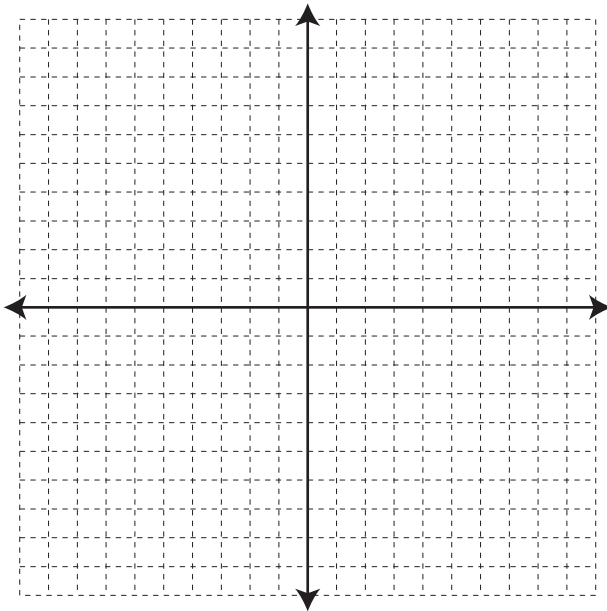
be 3, and in the line $y = -\frac{3}{4}x + 2$ the slope is $-\frac{3}{4}$.

We also discovered in the last class that the number that is added or subtracted to x is the point where the line crosses the y-axis (called the "y-intercept"). Using these two pieces of information, we can graph pretty much any line from the equation without making a table of values.

For example, to graph the line $y = \frac{1}{2}x + 3$, we notice two things:

1. The slope of the line is $\frac{1}{2}$, and
2. It crosses the y-axis at 3.

To graph it, just put your pencil at the y-intercept of $(0,3)$ and start making new points according to the slope. In this case, the rise is 1 and the run is 2, so move your pencil one square up (rise) and then two squares right (run) and put a new point. Then continue making new points and connect all the dots. You're done!



On your sheet of graph paper, graph each of the following.

1. $y = 4x - 1$

2. $y = -\frac{1}{2}x + 2$

3. $y = -2x + 5$

4. $y = -\frac{3}{4}x - 2$

5. $y = \frac{5}{6}x - 7$

6. $y = -\frac{3}{2}x - 2$

7. $y = \frac{5}{7}x - 4$

8. $y = 0x + 2$

9. $y = x - 2$

10. $y = -x + 8$